MINISTRY OF FINANCE, EGYPT.

Survey of Egypt.

Determination of the Exact Size and Orientation of the Great Pyramid of Gîza.

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Determination of the Exact Size and Orientation of the Great Pyramid of Giza.

In the beginning of April 1925 Prof. Borchardt of the German Institute of Egyptian Archæology asked the Survey of Egypt for the loan of surveying instruments in order to carry out a new accurate survey of the Great Pyramid of Gîza.

After consultation with Mr. R. Engelbach, Chief Inspector Antiquities Department, it was agreed that the Survey of Egypt would carry out the necessary measurements after Prof. Borchardt had done the necessary excavations and clearing to find the original base of the Pyramid on all four sides.

The construction of the Pyramid on the outside was as follows: The desert was cleared down to solid rock and on this rock was built a pavement which was accurately levelled. The actual base of the Pyramid was laid out on this pavement leaving about 40 centimetres width of pavement all round the bottom edge of the casing blocks. This width is, however, not exactly the same on all four sides, it being 38 centimetres on the western side, 42 centimetres on the northern side, and 48 centimetres on the eastern side, at the places where it could be measured. At the four corners of the Pyramid the rock was cut away, giving a greater depth for the foundations of these points. These excavations are rectangular in form and are called the corner sockets.

The purpose of this present survey is to determine as accurately as possible the exact size, shape and orientation of the original base of the Pyramid on the pavement.

METHOD OF SURVEY.

Frame Work.

Eight brass bolts were cemented into the rock round the base, one near each of the four corners, and, as these were not intervisible, four more were placed, one at about the middle of each side, in such a position that each point was visible from the adjacent points on each side of it.

These bolts are numbered from 1 to 8 clockwise from No. 1 which is at the S.E. corner of the Pyramid.

The positions of these points were determined by means of precise traverse observations.

The distances were measured with the Base Line Apparatus designed by M. M. Benoit and Guillaume using 24 metre standardized invar wires. Each length was measured twice with different wires with an accuracy of about 1 in 500,000.

The angles were measured on 4 arcs with a 6 inch Troughton and Simms micrometer theodolite.

The top of the flagstaff on the top of the Pyramid was visible at points 1, 6 and 7 and was included in the round of angles at these stations.

The closure in angle of this traverse was found to be 9·6″ which was adjusted by adding 1·2″ to each angle.

Using these adjusted angles, the co-ordinates were computed and a closing error of 8 millimetres in North direction and 0 millimetre in East direction was found. These co-ordinates were adjusted by the normal traverse method for closure.

The position of the top of the flagstaff was computed from the intersection of the sights at points 6 and 7. The computed horizontal angle at 1 to the top of the flagstaff and point 2 was found to check with the observed angle within 3·0″ of arc.

AZIMUTHS.

The azimuth of the line 6 to 7 was obtained by observations taken on Polaris near elongation against time. The final azimuth was the mean of two arcs taken both face left and face right. The time was taken on a Zenith watch rated to keep sidereal time.

The error of the watch was determined by vertical circle readings on both faces for both an East and a West star against time. The latitude of station 6 was computed from the known geodetic latitude of E 1, on the Great Pyramid, and the distance of 6 North of E 1.

The azimuth of 7 from 6 was found to be :—

119° 11′ 07·1″ East of true North.

EXCAVATION OF PAVEMENT.

North Side.—The whole of the pavement, where it existed on the North Side, was cleared and the original line of the edge of the casing blocks on it was found for a distance of 55 metres. On 20 metres of this, the casing blocks are still in position.

East Side.—An excavation was sunk down to the pavement, starting from the centre of the side and extending 30 metres to the north. In this

excavation the bottom edge of the casing blocks was found, giving a clear line for almost the entire length of the excavation.

SOUTH SIDE.—Five pits were dug at various places along the edge; in four of these it was found that the casing blocks and pavement had been destroyed. In one, the top edge of the casing block was still in place and the excavation was extended as far as possible along this edge, which was found for a distance of 18 metres, the edge being clearly defined for a distance of 15 metres.

WEST SIDE.—Four pits were sunk, the pavement being found in 3 of these.

In the fourth, the pavement had been entirely destroyed. The maximum length of the line given from these three holes was 28 metres.

PRECISE LEVELLING.

Before attempting to survey these lines, it was decided that it would be a useful check as to whether they really were on the pavement or not, if we determined the levels of each point.

Paint marks were made at various places on the pavement and casing-blocks, where they were found in the holes, and a line of precise levelling was run connecting each point with two Precise Bench Marks already established at the Pyramid. The descriptions of the points and their reduced levels above M.S.L. at Alexandria are given in the following list.

PRECISE LEVELLING AT GIZA PYRAMID.

B M.	Description of Points.	Value,
	North East Corner.	
1	Bottom of N.E. corner-socket	59·6958
2	Rock W. of B.M. 1=foundation of pavement	59·8913
	North Side.	
3	Top surface of pavement W. of 2 (beginning of pavement) at B. on diagram	60·4129
4	Top surface of pavement E. of remaining casing-blocks... ...	60·4049
5	Top surface of casing block (near 4)	61·9011
6	Top surface of pavement W. of remaining casing-blocks at A.	60·4115
7	Top surface of casing block (near 6)	61·9077
	North West Corner.	
8	Rock E. of N.W. corner socket=foundation of pavement...	59·8584
9	Botton of N.W. corner socket	59·6034
	West Side.	
10	Rock in hole L=foundation of pavement...	59·8841
11	Top surface of pavement in hole K	60·4129
12	Top surface of casing block in hole K...	61·9148
13	Top surface of pavement in hole J	60·4154
	South West Corner.	
14	Rock N. of S.W. corner socket=foundation of pavement ...	59·8798
15	Bottom of S.W. corner socket	59·8485
	South Side.	
16	Rock in hole H=top surface of pavement...	60·4222
17	Top surface of casing block in hole H	61·9333
18	Top surface of pavement in hole G	60·4229
19	Top surface of pavement in hole F	60·4264
	South East Corner.	
20	Bottom of S.E. corner socket	59·3700
21	Rock W. of S.E. corner sockeet=foundation of pavement ...	59·8305
	East Side.	
22	Top surface of pavement in the southern end of the hole C...	60·4193
23	Top surface of pavement in the northern part of the hole C...	60·4210

From this list we can see that the pavement is practically flat, but has a very slight slope of about 15 millimetres up from the N.W. corner to the S.E. corner.

MEASUREMENT OF THE BASE OF THE PYRAMID.

The method adopted for surveying the actual base of the Pyramid, *i.e.* the the lines of the bottom edge of the casing blocks on the pavement, was as follows :—

The 6 inch theodolite was set up on one edge of the excavation as near as possible in the line of the edge of the casing blocks still traceable on the pavement at the bottom of the excavation. It was then adjusted by means of the sliding head on the tripod until the marks on the extreme ends of the line on the pavement fell accurately on the centre hair of the telescope as it was revolved about its horizontal axis, the horizontal plate being level and clamped rigidly. A mark was then placed in the ground at the far end of the line, so that it also fell on the centre hair of the theodolite.

This mark was checked, using the opposite face of the theodolite.

The theodolite was set up over this mark, and after checking to make sure that the vertical hair would accurately traverse the line with the horizontal circle clamped, a second mark was placed on the ground, at the other end, in the continuation of the line. This mark was also checked on both faces.

The line between the two marks was extended in both directions by means of the theodolite, until it intersected the main traverse lines at each end.

A mark was placed in the ground at each point of intersection. This method was used on all four sides of the Pyramid, with a slight modification on the South side, since the top edge of the casing had to be taken, as the bottom edge was totally destroyed.

The whole work was repeated two or three times *ab initio*; the maximum difference in position of the final marks was about 3 centimetres, this being on the extension of the short southern edge. The mean position for each mark was taken as the best obtainable position.

The short distances between these marks and the nearest traverse points were measured with a standardized steel band graduated in centimetres and read to millimetres by estimation.

The co-ordinates of the points of intersection of the sides and the main traverse were found, and the equations of the four sides were written down. These equations were solved in pairs to give the co-ordinates of the corners of the Pyramid. From these co-ordinates the lengths and azimuths of all four sides and the two diagonals were found.

These are as follows :—

Side.	Length.	True Azimuth.		
	Metres.	°	′	″
North	230·253	89	57	32
South	230·454	89	58	03
East	230·391	359	54	30
West	230·357	359	57	30
N.E.—S.W.	325·699	44	56	45
N.W.—S.E.	325·868	314	57	03

The position of the South side was found from the actual prolongation of the whole of the existing top edge of the casing blocks, to which the bottom edge was assumed exactly parallel, and the horizontal distance of $\frac{5\cdot5}{7}$ times the vertical height of the edge above the pavement. This height is given in the list of levels, namely No. 17 to No. 16, and is 1·511 metres. Therefore the distance to be added is $1\cdot511\times5\cdot5/7=1\cdot187$ metres to the south of the line of the top of the casing. This distance has been added to get the co-ordinates of the south-east and south-west corners, and hence the actual lengths of the sides.

In justification of using this constant of 5·5/7, reference should be made to W. M. Flinders. *The Pyramids and Temples of Gîza* in which he gives the angle of slope as 51° 50′ 40″ ± 1′ 05″ also Borchardt, *Gegen die Zahlenmystik an der grossen Pyramide* ; finally a further check was made by measuring up the slope of the existing casing-stones on the North side. This gave approximately 1·173 metres as the horizontal distance for a difference in height of 1·496 metres (from points 4 to 5). The theoretical distance horizontally of 1·175 millimetres agrees within the limits of accidental error of the actual measurement.

The mean dimensions of the Pyramid are thus :—

Central Axis.	Length.	Azimuth.		
	Metres.	°	′	″
North to South	230·374	359	56	00
East to West	230·354	89	57	48
General Mean	230·364	—	3	06

These should be compared with Prof. W. M. Flinders Petrie's results found in 1880–1882 and published in the *Pyramids and Temples of Giza*. In this work, Prof. Petrie, after making accurate measurements and certain assumptions, obtains the following results :—

Side.	Length Petrie 1880	New Determination 1925.	Difference.
	Ins.	Ins.	Ins.
North	9069·4	9065·1	— 4·3
South	9069·5	9073·0	+ 3·5
East	9067·7·	9070·5	+ 2·8
West	9068·6	9069·2	+ 0·6
Mean	9068·8	9069·4	+ 0·6

From which we obtain the mean differences on the central axis :—

Axis.	Petrie minus Cole.
	Ins.
North to South	+ 1·7
East to West	— 0·4
Mean dimension ...	+ 0·6

This very close agreement of these mean dimensions shows the accuracy with which Prof. Flinders Petrie determined the data on which he based his assumptions.

The comparison of azimuths is as follows :—

Side.	Azimuth Petrie.		Azimuth Cole.		Difference.	
	′	″	′	″	′	″
N	— 3	20	— 2	28	+ 0	52
E	— 3	41	— 1	57	+ 1	44
S	— 3	57	— 5	30	— 1	33
W	— 3	54	— 2	30	+ 1	24
	— 3	43	— 3	06	+	37

These differences in azimuth are due to the fact that the new azimuths are found from the actual directions of the sides determined from the excavated pavement, whereas those of Prof. Petrie are of a hypothetical base obtained by computing " a square that shall pass through the points of the casing found on each side, and having also its corners lying on the diagonals of the sockets."*

In the lists of azimuths given above it will be seen that the most discrepant side is the East side which differs by about 3′ from the other three. In order to check this a further excavation was made on this side 23 metres south of the original one. In this excavation the pavement was found, and on it the line of the casing blocks was clearly shown. This line was found to lie exactly in the line as already extended, thus confirming the accuracy of the original extension.

Prof. Borchardt pointed out a small line on the pavement which projected a few centimetres from the edge of the casing-block about the middle of the North side. This line was neither a joint in the pavement nor in the line with the joint of the casing block. The measurements from the two northern corners to this line are as follows :—

to N.E. corner=115·161 metres.
to N.W. corner=115·090 ,,

Diff.= 71 millimetres.

Thus this line is probably the original line of the axis.

MAXIMUM ERRORS TO BE EXPECTED IN THE NEW DETERMINATION.

North Side.—This is the extension of a sharp and clearly defined line of 55 metres in length. The maximum error at either end should not exceed 6 millimetres.

East Side.—This is the extension of 52 metres of fairly clear line and is not likely to be in error by more than 6 millimetres at either end.

South Side.—This is the extension of 15 metres of the top edge of the casing blocks. The 15 metres chosen are unworn and give a very definite line to work from. The final extension should be correct to 1 centimetre on the West end and 3 centimetres on the East end, to which must be added any error there may be in the computed horizontal distance between the top and the bottom edges of the casing blocks.

* Petrie *op. cit.* p. 39.

WEST SIDE.—This is the extension of points in the pavement 28 metres apart. The pavement is badly worn in most places, as are also the casing blocks which are still in place. The points chosen are however fairly definite and there is little doubt that they are correct. The final errors at either end should be less than 3 centimetres.

It should be clearly understood that these possible errors are due to uncertainty of the absolute position of the line on the pavement since a difference of 1 millimetre only at either end of the line on the excavation will make a difference of 5 millimetres or more at each end of the extended side, to which must be added the possibility of slight deviation of the original side from the straight line. Checking along the 55 metres on the northern edge has shown this is likely to be inappreciable.

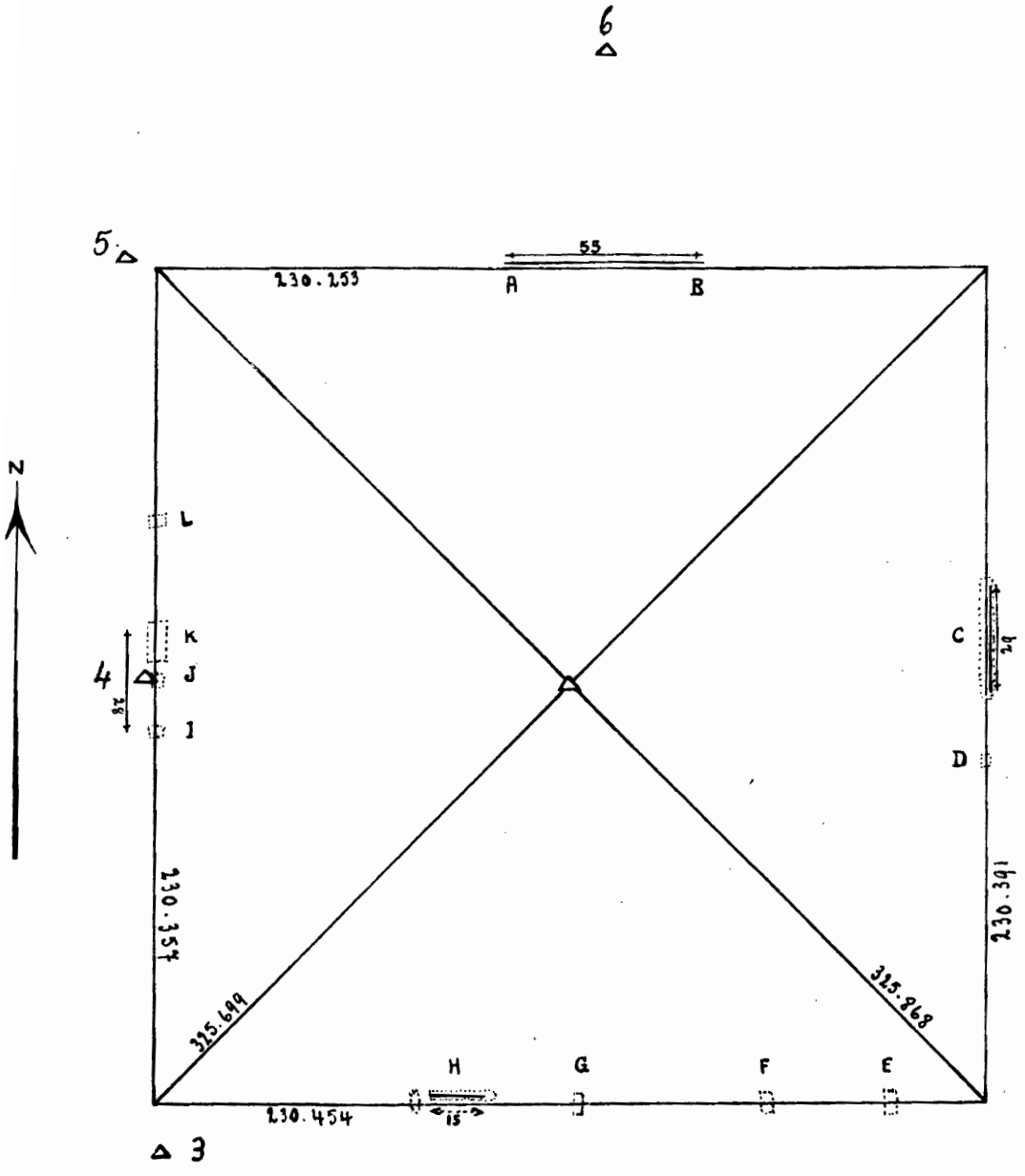
The position of the points of intersection of the adjacent sides was also found on the corner sockets, and measurements were taken from these points to the outside edges of the socket where they existed.

The South Western corner socket was too broken to give any information. For the other three sockets, the following measurements were found :—

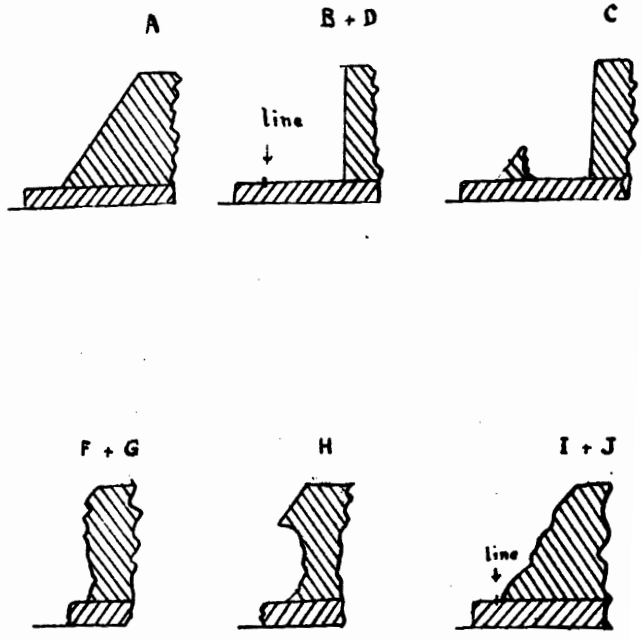
Socket.	Edge.	Distance.
		cms.
N.E.	N	85
	E	85
S.E.	S	83
	E	85
N.W.	N	75
	W	76

From these measurements we see that the corners of the Pyramid obtained from the extension of the sides actually fall on the diagonals obtained from the socket corners where these can be determined, thus giving additional confirmations that the size and orientation of each side as found above is very close to the truth.

SURVEY of the GREAT PYRAMID
GIZA



Rough - sketch of sections in various excavations.

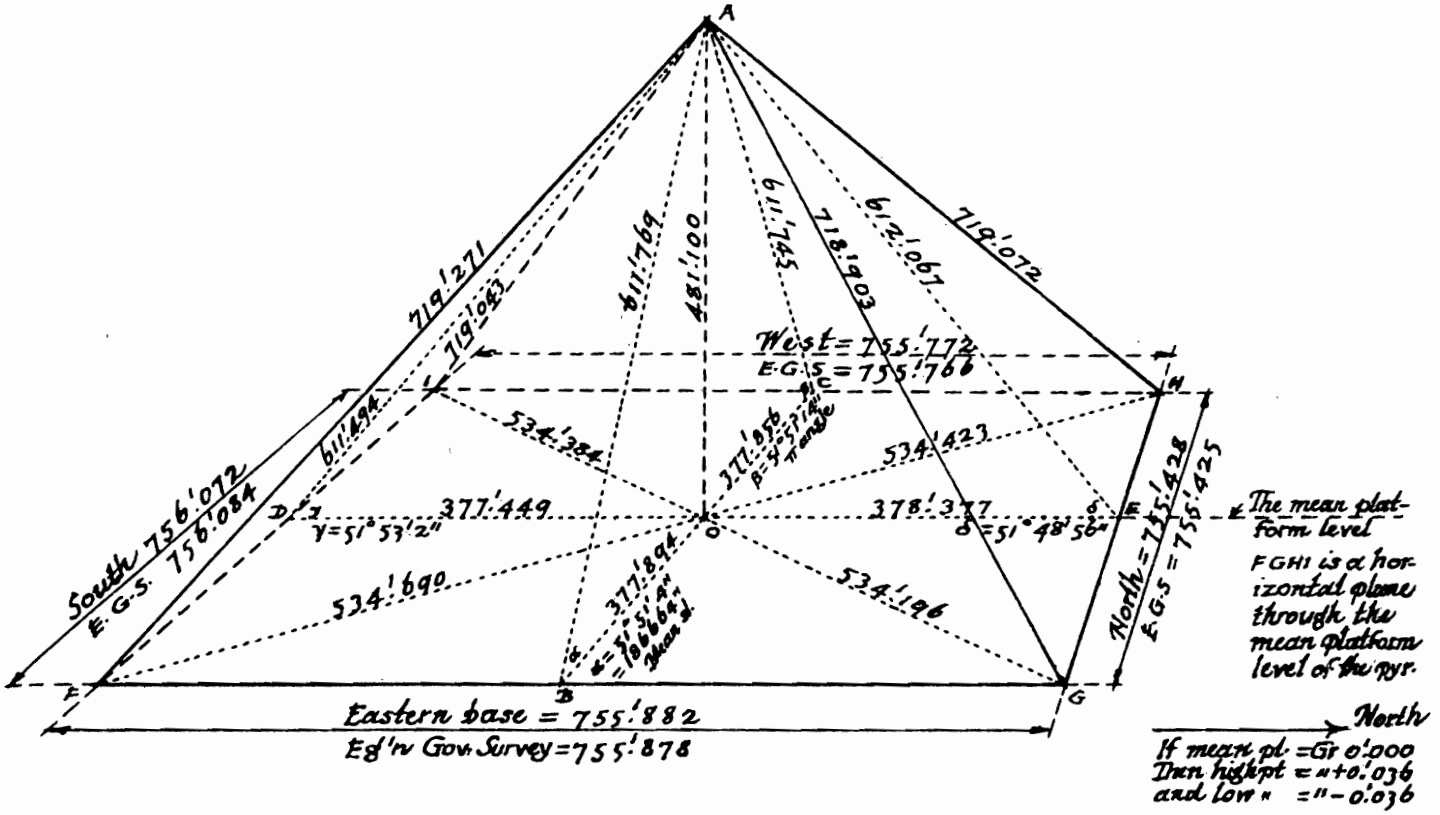


L + E the pavement is totally destroyed.

Scale 1 : 2,000

S.of E. 25/344.

The dimensions of the Great Pyramid of Gizeh based upon the surveys and measurements of Petrie and the Egyptian Government Survey of 1925. These are the dimensions that have been in this manuscript. From 9/12/33



The height and the slopes of the faces of the pyramid are based upon Petrie's data. His figure for the height of the pyramid is 481.133 ± 0.583.

The figures for the plan are based upon those of the Egyptian Government Survey of 1925. The dimensions of the latter are given in meters. For the conversion of these dimensions, from meters to feet, the mean of the 'legal' and 'international' meter length, namely, M = 3.2808473 has been used.

Lengths of the base sides

Taken at:	The Egyptian Gov't Survey	Variation
FG = The eastern base side ... = 755.1882	Measured 755.1878	The difference = +0.004
IH = " western " " ... = 755.1772	" 755.1766	" = +0.006
The mean of E and W. ... = 755.1827	" mean 755.1822	" = +0.005
GH = The northern base side ... = 755.1428	Measured 755.1425	" = +0.003
FI = " southern " " ... = 756.1072	" 756.1084	" = −0.012
The mean of N and S. ... = 755.1750	" mean 755.1755	" = −0.005

The slopes of the faces of the pyramid:

α	Taken at	51° 51' 4"	= 186664"	±	0.00"	= 186664"
β	"	51° 51' 14"	= "	+	1.0"	= 186667"
γ	"	51° 53' 2"	= "	+	1.18"	= 186782"
δ	"	51° 48' 56"	= "	−	1.28"	= 186536"

(See above) 00, 10, 18 and 28 are three terms of the Beta Series.

The ratio of expansion of the "Expanding Squares" = 100000 : 186664.

The so called π angle = 51° 51' 14"

OA, the height of the pyramid is equal to 4 3/4 mean polar and equitorial seconds of mer. latitude of the earth, (Duncan) ..... = 481.100

The perimeter of FGHI, the base of the pyramid = 30 seconds of mer. latitude at the equator .. = 3023.154

The mean of the south and west base sides = 440 Egyptian common cubits, var. 0.002 ..... = 755.1922

If a is the integral length, l.i.n, of ½ the mean base side, 378.000 and b is the integral height of the pyramid, r.t., 481.000, c the northern slant height, 612.066, and d is the mean of the east and west slant heights, 611.756, then $b^2/a = c$ and $\sqrt{a^2+b^2} = d$. Therefore, b is a mean proportional between a and c And if a is ½ the transverse axis of a hyperbola and b/2 the conjugate axis then c is ½ the latus rectum and d is the ax